Workshop on Complex Analytic and Algebraic Methods in Dynamics

15-18 January, 2019, at Osaka City University

Program

January 15

12:30 – 13:00 Registration

13:00 – 14:40 Takato Uehara (Okayama University)
Dynamical systems on complex surfaces

15:10 – 16:10 Martin Sera (Chalmers University of Technology and University of Gothenburg)
Products of currents with analytic singularities and its application

16:30 – 17:30 Takahiro Inayama (the University of Tokyo)
A converse of Hörmander's L^2 estimate and new positivity notions for vector bundles

January 16

10:20 – 12:00 Yutaka Ishii (Kyushu University)
Introduction to the dynamics of polynomial diffeomorphisms of $\mathbb{C}^2$

12:20 – 13:20 Shou Yoshikawa (the University of Tokyo)
Singular varieties admitting a polarized endomorphism

14:00– Discussion

January 17

10:20 – 12:00 Philipp Naumann (Bayreuth University)
An approach to Griffiths' conjecture

14:00 – 15:40 Yu Yasufuku (Nihon University)
Heights for orbits, and their applications to Zariski-density and integral points

16:10 – 17:10 Masanori Adachi (Shizuoka University)
On the normal bundle of Levi-flat real hypersurfaces

18:00– Banquet

January 18

10:20 – 12:00 Keiji Oguiso (the University of Tokyo)
Smooth projective surfaces with discrete and non-finitely generated automorphism group

12:20 – 13:20 Takahiro Shibata (Kyoto University)
On arithmetic degrees and canonical heights for self-morphisms on projective varieties

Abstracts

Masanori Adachi (Shizuoka University)
On the normal bundle of Levi-flat real hypersurfaces

A real hypersurface in a complex manifold is said to be Levi-flat if it is foliated by complex hypersurfaces. It is a natural question to explore the interaction between the dynamics of this foliation and function theory on the complement of the real hypersurface. This talk gives an introduction to problems concerning Levi-flat real hypersurfaces, emphasizing the role of the normal bundle of the foliation.

Takahiro Inayama (the University of Tokyo)
A converse of Hörmander's L^2 estimate and new positivity notions for vector bundles

Recently, Deng, Wang, Zhang, and Zhou established a new characterization of plurisubharmonic functions. They proved that an upper semi-continuous function satisfying "the Ohsawa-Takegoshi condition" is plurisubharmonic. This condition also corresponds to positivity notions for vector bundles. In my talk, I will explain their works and compare this new positivity with classical positivity, e.g. Griffiths positivity or Nakano positivity. I would also like to talk about a partial converse of Hörmander's L^2 estimate.

Yutaka Ishii (Kyushu University)
Introduction to the dynamics of polynomial diffeomorphisms of $\mathbb{C}^2$

My talk will be divided into two lectures. In the first lecture, I introduce some necessary terminologies from dynamical system theory and explain the motivation to study the dynamics of polynomial diffeomorphisms of $\mathbb{C}^2$. In the second lecture, I introduce the key notion "crossed mapping condition" and apply it to establish a criterion for hyperbolicity as well as to analyze the dynamics restricted to the real part $\mathbb{R}^2$.

Philipp Naumann (Bayreuth University)
An approach to Griffiths' conjecture

The Griffiths' conjecture asserts that every ample vector bundle over a compact complex manifold admits a hermitian metric with positive curvature in the sense of Griffiths. Apart from the case of curves and of course line bundles, this conjecture is completely open. In the talk we give an approach to this problem by using curvature formulas for direct images and the relative Kähler-Ricci flow.

Keiji Oguiso (the University of Tokyo)
Smooth projective surfaces with discrete and non-finitely generated automorphism group

According to Professor Igor Dolgachev, it has been a long standing problem if there is a smooth projective surface such that the automorphism group is discrete and non-finitely generated and/or it has infinitely many non-isomorphic real forms.

In this talk, I would like to show the following answers:

1) There is a smooth complex projective surface, birational to some K3 surface, such that the automorphism group is (discrete and) non-finitely generated and with infinitely many non-isomorphic real forms (joint work with Professor Dinh).

2) Over k, there is a smooth projective surface birational to some K3 surface, such that the automorphism group is (discrete and) non-finitely generated.

3) Over k_0, there is not a smooth projective surface birational to some K3 surface, such that the automorphism group is (discrete and) non- finitely generated.

Here k (resp. k_0) is any algebraically closed field of odd characteristic of transcendental degree ≥ 1, possibly infinite, (resp. 0) over the prime field.

Martin Sera (Chalmers University of Technology and University of Gothenburg)
Products of currents with analytic singularities and its application

Since the fundamental results of Bedford and Taylor, the wedge product of closed positive currents

is a central tool in complex analysis. Demailly generalized these results for currents with unbounded potentials under the assumption that the degree is small enough. To consider the wedge product of currents in arbitrary degrees, there appeared several different approaches which turned out to be not equivalent. Andersson and Wulcan presented a definition of $(dd^c\varphi)^k$ (also called Monge Ampère operator) for arbitrary k and for plurisubharmonic functions φ with analytic singularities. Together with Błocki, they proved a mass formula for these MA operators. In a joint work with R. Lärkäng, H. Raufi & E. Wulcan, we extend the definition from above to wedge products of closed positive (1,1) currents with mixed potentials. We show that these coincide with the wedge products given by Bedford-Taylor-Demailly when they are defined. Furthermore, we present a definition for mixed Monge Ampère operators involving correction terms inspired by the mass formula mentioned above. We obtain that these so-called generalized MA products behave well under push forward. In particular, for a Griffiths semipositive singular Hermitian metric (with analytic singularities) on a vector bundle, we can define singular Chern forms which represent the Chern classes of the vector bundle.

Takahiro Shibata (Kyoto University)
On arithmetic degrees and canonical heights for self-morphisms on projective varieties

Let us consider a projective variety over a number field and a morphism from the variety to itself. Then our main interest is the growth of the height of a point along the iteration of the action of the morphism to the point. The notion of arithmetic degree is a quantity which measures the growth. Canonical height functions are normalizations of usual height functions which reflects the dynamics of the morphism. In this talk, I will explain these notions, relationships between them, and related conjectures and results.

Takato Uehara (Okayama University)
Dynamical systems on complex surfaces

The entropy measures the complexity of a dynamical system, and if the entropy is positive, the system exhibits a chaotic behavior. In this talk, we consider the case where dynamical systems are given by automorphisms on compact complex surfaces, and investigate these dynamical systems in terms of entropy values.

Yu Yasufuku (Nihon University)
Heights for orbits, and their applications to Zariski-density and integral points

Loosely speaking, arithmetic dynamics studies arithmetic properties of iterations of self-maps on algebraic varieties (defined over number fields, finite fields, p-adic fields, function fields, etc.). One important tool is height function, including Weil height, canonical height, and local height. In this survey talk, we will first introduce height functions, and then discuss how heights are used in theorems and conjectures in arithmetic dynamics, especially focusing on Zariski-density and integral points.

Shou Yoshikawa (the University of Tokyo)
Singular varieties admitting a polarized endomorphism

To study singular varieties admitting an endomorphism is very important for study of smooth varieties admitting an endomorphism. For example, a polarized endomorphism induces the endomorphism of the affine cone, but it has bad singularities in general. Moreover, running MMP for the smooth variety admitting a polarized endomorphism, this endomorphism induces the endomorphism of varieties appearing in any step of MMP, but in general, the varieties are not smooth. Affine cones and MMP are crucial tools for classification of projective varieties.

First, we prove that the bad singular locus is totally invariant, so the global endomorphism induces the local endomorphism at the generic point of this locus. Furthermore, we introduce related results.

Next, we consider singular surfaces admitting a polarized endomorphism by using MMP. It is very useful by studying 3-folds admitting a polarized endomorphism.