Geometric Complex Analysis on Foliations and Dynamics

25-27 November, 2019, at RIMS.[1]

Program

November 25

12:30 - 13:00 Registration

13:00 - 14:00 Tetsuo Ueda (Kyoto University)
Plurisubharmonic functions and foliations on affine line bundles over compact Kähler manifolds

14:30 - 15:30 Laurent Stolovitch (Universite Cote d'Azur)
Linearizations of neighborhoods of embedded of complex compact manifolds and foliations

16:00 - 17:00 Jun-Muk Hwang (KIAS)
Infinitesimal neighborhoods of submanifolds

November 26

9:00 - 10:00 Takeo Ohsawa (Nagoya University)
$\mathbb{C}^n$ and $\mathbb{CP}^n$ in analytic families

10:15 - 11:15 Noboru Ogawa (Tokai University)
On the neighborhood of a torus leaf and dynamics of holomorphic foliations

11:30 - 12:30 Yusuke Okuyama (Kyoto Institute of Technology)
A generalization of the converse of Brolin's theorem

14:00 - 15:00 Takato Uehara (Okayama University)
Siegel disks for rational surface automorphisms with positive entropy

15:30 - 16:30 Shin-ichi Matsumura (Tohoku University)
On projective manifolds with semi-positive holomorphic sectional curvature

November 27

9:00 - 10:00 Masanori Adachi (Shizuoka University)
Sobolev estimates for the complex Green operator on Levi-flat manifolds II

10:30 - 11:30 Shinichi Tajima (Niigata University)
On the Suwa method for computing versal unfoldings of codimension one complex analytic singular foliations

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Abstracts

Tetsuo Ueda (Kyoto University)
Plurisubharmonic functions and foliations on affine line bundles over compact Kähler manifolds

We study function theoretic properties of holomorphic $\mathbb{C}$-bundles X over compact Kähler manifolds M in the case where the (real) first Chern class of the associated line bundle vanishes. We give a precise restriction from below on the growth rate of nonconstant plurisubharmonic functions on X. This is proved by utilizing a real analytic foliation with holomorphic leaves on X. If M is of dimension 1, there exists a strictly plurisubharmonic exhaustion function on X of minimal growth. On the other hand, if M is of dimension ≥ 2, then X is not Stein. This talk is based on a joint work with T. Koike.

Laurent Stolovitch (Universite Cote d'Azur)
Linearizations of neighborhoods of embedded of complex compact manifolds and foliations

In this work, we address the following question due to Grauert: Let C be complex compact manifold embedded into two complex manifolds M and N. Assume a neighbourhood of C in M is formally equivalent to a neighborhood of C in N. Are they biholomorphically equivalent ? If C is embedded in M, then it is also embedded, as the zero section, into the normal bundle of C in M. We shall give conditions that ensure that a neighborhood of C in the first one is biholomorphic to a neighborhood of C int the second. We also prove the existence of a holomorphic foliation in M having C as a compact leaf, extending Ueda ' s theory to the high codimension case. The solution to this problem involves "small divisors problems". This is joint work with X. Gong (Univ. Wisconsin-Madison).

Jun-Muk Hwang (KIAS)
Infinitesimal neighborhoods of submanifolds

We discuss the rigidity problem of infinitesimal neighborhoods of compact submanifolds of complex manifolds, which goes back to a question of Nirenberg and Spencer asking whether a finite-order infinitesimal neighborhood determines the germ of a submanifold when the normal bundle is positive. To study the problem for a larger class of submanifolds, we reformulate the problem in terms of families of infinitesimal neighborhoods of submanifolds. We explain some affirmative results on this reformulated problem.

Takeo Ohsawa (Nagoya University)
$\mathbb{C}^n$ and $\mathbb{CP}^n$ in analytic families

A complex manifold M with a holomorphic map π from M onto a complex manifold T is called an analytic family if $d\pi \neq 0$ and $M_t := \pi^{-1}(t)$ $(t \in T)$ are all homeomorphic to each other. Results on analytic families with $M_t \cong \mathbb{C}^n$ or $M_t \supset \mathbb{CP}^n$ will be reviewed.

Noboru Ogawa (Tokai University)
On the neighborhood of a torus leaf and dynamics of holomorphic foliations

We discuss the relation between semi-local dynamics of the holonomy along a compact leaf L of a holomorphic foliation $\mathcal{F}$ (non-singular, codimension one) and Ueda's classification of the complex analytic structure on a neighborhood of L. In this talk, we focus on the case that L is an elliptic curve. This is joint work with T.Koike.

Yusuke Okuyama (Kyoto Institute of Technology)
A generalization of the converse of Brolin's theorem

For a rational function on the Riemann sphere, if its degree is more than 1, then there is the unique balanced probability measure on the Riemann sphere which has no mass on the exceptional set. When this rational function is a polynomial, Brolin constructed such a unique balanced probability measure as the harmonic measure of the Julia set with pole at infinity. In this talk, we establish a generalization

of the converse of Brolin's theorem, including a potential theoretic characterizaion of polynimials among rational functions, in an optimal form. This talks is based on a joint work with Małgorzata Stawiska.

Takato Uehara (Okayama University)
Siegel disks for rational surface automorphisms with positive entropy

In this talk, we consider rational surface automorphisms preserving reduced anticanonical curves, and discuss when and where Siegel disks appear under the assumption that these automorphisms have positive entropy. Moreover, we show that there exists an automorphism with a given number of Siegel disks.

Shin-ichi Matsumura (Tohoku University)
On projective manifolds with semi-positive holomorphic sectional curvature

In this talk, I explain the geometry of a projective manifold (more generally a Kaehler manifold) X with semi-positive holomorphic sectional curvature. I first show that, if X has positive holomorphic sectional curvature, then X is rationally connected, that is, arbitrary two points can be connected by a rational curve (the image of $\mathbb{P}^1$ by a holomorphic map). This result gives an affirmative solution for Yau's conjecture (which was proved by Yang). Moreover, I show that, if X has semi-positive holomorphic sectional curvature, X admits a locally trivial morphism from X to Y such that the fiber F is rationally connected and the image Y has a finite etale cover by an abelian variety A. This structure theorem can be seen as a generalization of the structure theorem proved by Howard-Smyth-Wu and Mok for holomorphic "bisectional" curvature. The proof depends on the theory of holomorphic foliations. MRC fibrations, and singular hermitian metrics.

Masanori Adachi (Shizuoka University)
Sobolev estimates for the complex Green operator on Levi-flat manifolds II

In function theory on Levi-flat manifolds, the transverse regularity of leafwise holomorphic functions is a subtle problem. It was shown by Ohsawa and Sibony that a compact Levi-flat manifold equipped with a leafwise positive line bundle enjoys certain Sobolev estimate of finite order for the complex Green operator, and it can be embedded in a projective space by a leafwise holomorphic map with finite transverse regularity. In this talk, we shall refine the estimate of Ohsawa and Sibony by using the d'Angelo 1-form, and show that the refined estimate is sharp in general by giving an example where better estimates must fail.

Shinichi Tajima (Niigata University)
On the Suwa method for computing versal unfoldings of codimension one complex analytic singular foliations

In a series of papers, T. Suwa investigated singularities of holomorphic foliations and developed a theory of unfoldings. He gave in particular a method for computing first order versal unfoldings of codimension one local foliations and showed that a first order versal unfolding is versal. As the method given by Suwa involves several computations in local rings and modules, it is difficult in general to perform required computation in practice. In this talk, we consider the Suwa method from the point of view of computational complex analysis. Based on the Suwa method, we describe an algorithm for computing versal unfoldings of singular foliations. The key of our approach is the use of Grothendieck local duality on residues.