Studies on Pseudoconvexity of General Order

大阪市立大学, 2020/3/8-10[1]

Program

March 8

9:50 – 10:50 Masanori Adachi (Shizuoka)
The Diederich–Fornaess index and its variants

11:00 – 11:50 Makoto Abe (Hiroshima)
A Characterization of Subpluriharmonicity for a Function of Several Complex Variables

13:30–15:00 Thomas Pawlaschyk (Wuppertal)
On q-plurisubharmonic functions in $\mathbb{C}^n$

15:30 – 16:30 Jihun Yum (Busan)
Diederich–Fornaess index and Steinness index for abstract CR-manifolds

March 9

10:00 – 11:30 Thomas Pawlaschyk (Wuppertal)
On q-pseudoconvex domains in $\mathbb{C}^n$

13:30 – 14:30 Makoto Abe (Hiroshima)
Intermediate pseudoconvexity for unramified Riemann domains over $\mathbb{C}^n$

14:40 – 15:40 Kazuko Matsumoto (Tokyo University of Science)
On the theorems of Rothstein and Sperling concerning continuations of analytic sets

15:50 – 16:50 Shun Sugiyama (Hiroshima)
A new proof of theorem of Eastwood–Vigna–Suria

March 10

9:00 – 10:30 Thomas Pawlaschyk (Wuppertal)
On q-pseudoconcave sets in $\mathbb{C}^n$

10:40 – 11:30 Takeo Ohsawa (Nagoya)
Extension problems and notions of convexity

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Abstracts

Masanori Adachi (Shizuoka)
The Diederich–Fornaess index and its variants

The Diederich–Fornaess index is a classical notion to measure the strength of pseudoconvexity for weakly pseudoconvex domains and known to have applications in the L^2 (Sobolev) esimates for the dbar operator. This talk shall give an overview on recent developments around this index including its estimates (by Levi rank, D'Angelo 1-form, or other geometric conditions) and its variants (for pseudoconcavity, rough boundary, or CR manifolds). In particular, we shall illustrate some of the results for the complements of Levi-flats.

Makoto Abe (Hiroshima)
A Characterization of Subpluriharmonicity for a Function of Several Complex Variables

We give a characterization of a subpluriharmonic function of several complex variables in the sense of Fujita (J. Math. Kyoto Univ., **30**: 637–649, 1990) by using polynomial functions of degree at most two. This is joint work with Shun Sugiyama.

Thomas Pawlaschyk (Wuppertal)
On q-plurisubharmonic functions in $\mathbb{C}^n$ (March 8)

I will talk about upper semi-continuous q-plurisubharmonic functions in the sense of Hunt-Murray and compare with other notions of generalized convex functions. I will present different characterizations of q-plurisubharmonic functions and show approximation techniques.

Jihun Yum (Busan)
Diederich–Fornaess index and Steinness index for abstract CR-manifolds

Let Ω be a smooth bounded pseudoconvex domain in $\mathbb{C}^n$. The Diederich–Fornaess index and the Steinness index of Ω are defined by

$$DF(\Omega) := \sup_{\rho}\{0 < \eta < 1 : -(-\rho)^{\eta} \text{ is strictly plurisubharmonic on } \Omega\},$$

$$S(\Omega) := \inf_{\rho}\{\eta > 1 : \rho^{\eta} \text{ is strictly plurisubharmonic on } \overline{\Omega}^{\complement} \cap U \text{ for some neighborhood } U \text{ of } \partial\Omega\}.$$

Roughly speaking, $DF(\Omega)$ is the supremum of the Hölder exponents of these exhaustions near the boundary, and $S(\Omega)$ is the infimum of the Hölder exponents of positive strictly plurisubharmonic functions in $\overline{\Omega}^{\complement}$ which approaches to zero on the boundary $\partial\Omega$. We first describe two indices in terms of a special 1-form on $\partial\Omega$, called the D'Angelo 1-form. Then we will see how this characterization allows us to define two indices for abstract CR-manifolds of hypersurface type.

Thomas Pawlaschyk (Wuppertal)
On q-pseudoconvex domains in $\mathbb{C}^n$ (March 9)

I will present a list of different characterizations of q-pseudoconvex domains which can be defined by, e.g., q-plurisubharmonic exhaustion functions. Furthermore, I will give examples of q-pseudoconvex domains.

Makoto Abe (Hiroshima)
Intermediate pseudoconvexity for unramified Riemann domains over $\mathbb{C}^n$

We characterize the q-pseudoconvexity for unramified Riemann domains over $\mathbb{C}^n$, where $1 \leq q \leq n$, by the continuity property which holds for a class of maps whose projections to $\mathbb{C}^n$ are families of unidirectionally parameterized q-dimensional analytic balls written by polynomials of degree at most two. This is joint work with Tadashi Shima and Shun Sugiyama.

Kazuko Matsumoto (Tokyo University of Science)
On the theorems of Rothstein and Sperling concerning continuations of analytic sets

We generalize some theorems of Rothstein–Sperling (1966) concerning continuations of analytic sets by using of the notions of pseudoconvex domains of general order (or equivalently, locally q-complete domains with corners in the sense of Diederich–Fornaess).

The results are as follows.
(1) Let D be a bounded domain of pseudoconvex of order k $(0 \leq k \leq n-1)$ in $\mathbb{C}^n$ $(n \geq 2)$ and let S be an irreducible analytic set of dimension m in a neighborhood of the boundary ∂D. If $m \geq n-k+1$, then S is continuable to an analytic set in a neighborhood of $\overline{D}$.
(2) In addition to (1), suppose that the domain D is defined by $D := \{z \in \mathbb{C}^n : \varphi(z) < 0\}$ for some $\varphi\colon \mathbb{C}^n \to \mathbb{R}$ which is pseudoconvex exhaustion function of order k. If $m \geq \max(n-k, 2)$, then S is continuable to an analytic set in a neighborhood of $\overline{D}$.
(3) The bounds of dimension m of S in (1) and (2) are best.
This is a joint work with O. Fujita (1931-2018). The three examples in (3) are due to him only.

Shun Sugiyama (Hiroshima)
A new proof of theorem of Eastwood–Vigna–Suria

In this talk, I will explain the relation between q-pseudoconvexity and an algebraic condition of cohomology groups. Especially, I will introduce a new proof of theorem of Eastwood–Vigna–Suria. This proof is based on Kajiwara-Kazama's method in 1973.

Thomas Pawlaschyk (Wuppertal)
On q-pseudoconcave sets in $\mathbb{C}^n$ (March 10)

The q-pseudoconcave sets are closed sets in $\mathbb{C}^n$ whose complement is q-pseudoconvex. They occur naturally as analytic sets of certain minimal dimension. On the converse, one may ask whether a closed q-pseudoconcave set A admits a local foliation by complex submanifolds. I will give a positive answer in the case when A is the graph of a continuous mapping $f\colon \mathbb{C}^n \times \mathbb{R} \to \mathbb{R} \times \mathbb{C}^p$.

Takeo Ohsawa (Nagoya)
Extension problems and notions of convexity

Extension theorems from complex analytic subsets play important roles in several complex variables. We shall recall how the solvability of the d-bar equation with L^2 estimates leads to extension theorems and report recent results on a rigidity question. We shall also discuss generalizations to q-convex manifolds, based on my paper published in 2007.